Proof of Beal-Conjecture & Nature of Beal equation with respect to its exponents when bases are prime to each other

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Abstract

This paper contains the proof of Beal-conjecture. In my first publication in August 2013 and in subsequent developments thereafter through publication of papers in so many bouts, I have been claiming the proof with the saying of following facts and figures.
Sum or difference of two odd numbers (say a & c) is an even number (say b), (a, b, c) = 1, where a produces power by virtue of Nd-operation and c produces power by virtue of Ns-operation. B produces power completely in separate manner which will be shown in this paper. Earlier I claimed that a & c both cannot be in power form as Nd & Ns operation cannot run simultaneously. But it is not true. Fact is that when any two elements produce power beyond two, other element must be restricted to two from producing power. It is possible when and only when we allow a common factor among a, b, c. This paper also contains the nature of exponents when all elements are in power form beyond two.

Key words:

N-equation & Nize-equation, positive & negative wings, unit wing, N_d & N_s– operation, zygote elements

Introduction:

I believe that I have been able to give a solid proof in favour of Beal equation mystery. Over the past ten years I have been trying to develop a theory i.e. named by me as 'Wings-theory'. It mainly deals with the expressions of sum or difference of two square integers. With the help of this theory it has been possible to arrange all the Pythagorean triplets in a systematic manner and to establish the formulae for the power formation of two odd elements on both the sides of Pythagorean relation $a^2 + b^2 = c^2$ and so many other properties of wings. This theory has been published in 'IJSER Journal, Houston, USA' in different phases among which the last published papers in May, 2017 can be referred to this present paper. Only the power formation part of this published paper is relevant here, based on which this simple & sound proof of Beal equation has been possible to be projected.

1. Some fundamental related theorems and definitions:

a) The expressions $w = u^2 \pm v^2$ where $(u, v) = 1$ are said to have two wings, one is positive and other is negative. In general u, v are the combinations of odd & even integers unless it is specially mentioned or clearly understood. If $u – v = 1$ it can be specially called unit wing, positive or negative.

b) Prime numbers can be divided into two categories, 1st kind and 2nd kind depending upon $4x – 1$ form or $4x + 1$ form respectively.

c) P^n where $P \in$ prime has a single negative unit wing i.e. difference of two squares of even or odd integers (consecutive) i.e. $\{(P^n + 1)/2\}^2 – \{(P^n – 1)/2\}^2$

d) P^n where $P \in 2^{nd}$ kind odd prime has a single positive wing, may be unit or may not be.

e) $N = P_1^m.P_2^n$ where P_1 & P_2 are two primes, produces two negative wings, one is unit i.e. $\{(N + 1)/2\}^2 - \{(N - 1)/2\}^2$ and other is non-unit (by N_d-operation)

f) $N = P_1^m.P_2^n$ also produces two positive wings, may be unit or may not be but both $P_1, P_2 \in 2^{nd}$ kind only (by N_s-operation)

N_d & N_s are two operations to produce negative & positive wings as defined in the referred paper vide page no. 137

g) Total number of negative or positive wings of $N = P_1^{n1}P_2^{n2}.....P_k^{nk}$ is 2^{k-1}

h) The Pythagorean relation $a^2 + b^2 = c^2$ where $a, b, c \in I^+$ & $(a, b, c) = 1$ is called Natural equation or simply N-equation when its compared equation $(a_0^2 - b_0^2)^2 + (2a_0b_0)^2 = (a_0^2 + b_0^2)^2$ has the zygote elements a_0 & b_0 of rational & integer values. Generally a & c denote odd elements. When zygote elements both are not rational integers, relation is called Niz-equation

i) For unit negative wing in N-equation $a^2 = c^2 - b^2$, obviously c & b both cannot be in power form as $\alpha^m - \beta^n \neq 1$ so long $m, n > 1$, nor c, b can be the combination of rational & irrational nor c, b can be both irrational.

2. $a^{2p+1} \pm b^{2q+1} = c^{2r+1}$ where $p, q, r > 0$, $a, c \in$ odd integers, has no existence.

Say, any odd number $N = xy = \alpha^{2n+1} + \beta^{2n+1} = (\alpha + \beta)(\alpha^{2n} - \alpha^{2n-1}\beta + \alpha^{2n-2}\beta^2 - \ldots\ldots + \beta^{2n})$ where $\alpha + \beta = x$

$\Rightarrow \alpha^{2n} - (x - \alpha)\alpha^{2n-1} + (x - \alpha)^2\alpha^{2n-2} - \ldots\ldots. + (x - \alpha)^{2n} = y$

$\Rightarrow (2n + 1)\alpha^{2n} - xk_1\alpha^{2n-1} + x^2k_2\alpha^{2n-2} - \ldots\ldots. + (x^{2n} - y) = 0$ i.e. $f(\alpha) = 0$ which will produce pair wise integer or non-integer roots like $(\alpha_1, x - \alpha_1)$, $(\alpha_2, x - \alpha_2)$, $(\alpha_3, x - \alpha_3)$, ………. to have the relations $\alpha_1^{2n+1} + (x - \alpha_1)^{2n+1} = \alpha_2^{2n+1} + (x - \alpha_2)^{2n+1} = \alpha_3^{2n+1} + (x - \alpha_3)^{2n+1} = \ldots\ldots$ considering all integer roots among all possible cases of N to express as a product of two numbers xy.

e.g. $1729 = 7.13.19 = 1^3 + 12^3 = 9^3 + 10^3$ where $a + b = 13$ & $a + b = 19$

Here, we may get relations like $\alpha_1^{2n+1} + (x - \alpha_1)^{2n+1} = \alpha_2^{2m+1} + (x - \alpha_2)^{2m+1}$ but due to presence of free constant term $(x^{2n} - y)$ we will never get one root = 0.

$\Rightarrow$ There is no existence of $\alpha_1^{2n+1} + (x - \alpha_1)^{2n+1} = \alpha_2^{2m+1}$

It further confirms the proof that $p^n = a^m \pm b^r$ does not have any existence where $p \in$ odd prime & all $n, m, r \in$ odd.

Obviously, if $(m, r) = q$ then $p^n = (a_1)^q + (b_1)^q$ which has no existence (already proved)

Again, if $(m, r) = 1$, then $p^n = a^m + (b^{r/m})^m$ where $a \pm b^{r/m}$ is an irrational and cannot be equal to 1 considering p^n as a product of two factors $p^n.1$

In place of p if it is an odd integer same logic can be applied. A mixed irrational number cannot be equated with a rational factor of a number.

3. Proof of Beal Conjecture: the equation $a^x + b^y = c^z$ where all variables are positive integer & $x, y, z > 2$ and $(a, b, c) = 1$, has no existence.

In N-equation $a^2 + b^2 = c^2$ where a & c are odd elements, 'a' element produces power as per the formula

$(\alpha^2 - \beta^2)^n \pm \{{}^nc_1\alpha^{n-1}\beta + {}^nc_3\alpha^{n-3}\beta^3 + \ldots\ldots\}^2 = \pm \{\alpha^n + {}^nc_2\alpha^{n-2}\beta^2 + \ldots\ldots\}^2$ …………**(A)**

If a is in the form of $4\lambda - 1$ & produces odd power then we receive the negative sign resulting the relation

$a^n + c^2 = b^2$

c is always in the form of $4\lambda + 1$ and produces power as per the formula

$\{\alpha^n - {}^nc_2\alpha^{n-2}\beta^2 + \ldots\}^2 + \{{}^nc_1\alpha^{n-1}\beta - {}^nc_3\alpha^{n-3}\beta^3 + \ldots\}^2 = (\alpha^2 + \beta^2)^n$ ………… **(B)**

Say, eq. (A) is $a^n + u^2 = v^2$ where obviously $v + u = (\alpha + \beta)^n$ i.e. one of the complete factors of a^n (with full exponent) & $v - u = (\alpha - \beta)^n$ i.e. one of the complete factors of a^n.

Now, say $a^n = d_1{}^n.d_2{}^n$ where d_1 & d_2 are two primes.

$\Rightarrow d_1{}^n = \{(d_1{}^n + 1)/2\}^2 - \{(d_1{}^n - 1)/2\}^2$ and $d_2{}^n = \{(d_2{}^n + 1)/2\}^2 - \{(d_2{}^n - 1)/2\}^2$

By N_d-operation it will produce two negative wings, one is unit and other is non-unit and it will have no other additional wings.

$\Rightarrow (d_1d_2)^n = \{(d_1{}^n + 1)(d_2{}^n + 1)/4 \pm \{(d_1{}^n - 1)(d_2{}^n - 1)/4\}^2 - \{(d_1{}^n + 1)(d_2{}^n - 1)/4 \pm \{(d_2{}^n + 1)(d_1{}^n - 1)/4\}^2$

$\Rightarrow (d_1d_2)^n = \{(d_1{}^nd_2{}^n + 1)/2\}^2 - \{(d_1{}^nd_2{}^n - 1)/2\}^2$ and $= \{(d_2{}^n + d_1{}^n)/2\}^2 - \{(d_2{}^n - d_1{}^n)/2\}^2$

Obviously 2nd one is non-unit wing i.e. $(d_1d_2)^n = \{(d_2{}^n + d_1{}^n)/2\}^2 - \{(d_2{}^n - d_1{}^n)/2\}^2$

Say, both the RH elements are in power form s^p & t^q.

$\Rightarrow (d_1d_2)^n = (s^p)^2 - (t^q)^2$ where obviously $d_2{}^n = s^p + t^q$ & $d_1{}^n = s^p - t^q$ considering $d_1 < d_2$

If we consider p & q both are even i.e. $d_1{}^n = s^{2p} - t^{2q}$, we get two equations as d_1 is a prime

$s^p - t^q = 1$ ……… (c) & $s^p + t^q = d_1{}^n$ ………(d) where $p, q > 1$

But for $p, q > 1$ eq. (c) does not exist & if (c) does not exist, eq. (d) has also no existence.

Because, $(s^p + t^q) = d_1{}^n/(s^p - t^q)$.

Now, if d_1 & d_2 are itself in different power form i.e. $a^n = (d_1{}^{m1}.d_2{}^{m2})^n$, it does not matter as because both $d_1{}^{m1}$ & $d_2{}^{m2}$ possess a single negative unit wing. It is all the same whether we consider $d_1{}^{m1}$ or ignore m_1.

If $a = d_1d_2d_3\ldots d_k$ which will produce 2^{k-1} number of negative wings, same logic can be applied on any one of the wings to isolate a separate group having less number of elements ($k_1 < k$) held responsible to produce both elements in power form. This logic can be again repeated on this separate group to isolate another group ($k_2 < k_1$) and so on to achieve finally an isolated prime where two non-existent relations come into picture.

$\Rightarrow$ If we consider that a^n produces a negative wing where both elements in power form then at the end of chain-process we have to accept two non-existent relations.

Here lies the proof of Beal conjecture.

The proof also covers the relation $d_1{}^x + d_2{}^y = e^z$ where d stands for odd & e for even & $x, y, z > 2$ as $d_1{}^n + d_2{}^2 = e^2$ is a particular form of N-equation under (A).

The proof can be illustrated by a simple example:

Say, $N = (3.5.7.211)^5$ which produces $2^{4-1} = 8$ numbers of negative wings where one of the wings produces two elements in power form beyond 2 (may be called special wing)

$\Rightarrow$ any combination less than the rest by magnitude can be considered as an isolated factor on which condition applies to produce a special wing.

Here, 4 wings will be received from (combination of three)^5 versus (rest)^5 in descending order where isolated factor may be 3^5 or 5^5 or 7^5 or $(3.5.7)^5$, one of which will produce a special wing.

3^5, 5^5, 7^5 cannot produce that special wing. $\Rightarrow (3.5.7)^5$ may produce that special wing. If $(3.5.7)^5$ produces that special wing, any one of 3^5 or 5^5 or 7^5 has to produce that special wing which is absurd.

Now, another set of 3 wings will be received by (combination of two)^5 versus (rest)^5 in descending order where isolated factors are $(3.5)^5$ or $(3.7)^5$ or $(5.7)^5$ which will produce that special wing. Take any one say, $(5.7)^5$. It will produce a special wing if 5^5 produces a special wing which is absurd.

The 8th wing obviously is an unit negative wing i.e.$\{(N + 1)/2\}^2 - \{(N - 1)/2\}^2$ which cannot produce a special wing.

Hence, both for N-equation where zygote expression are rational & for Niz-equation where zygote expression are irrational nature, $a^x + b^y = c^z$ where x, y, z > 2, cannot exist for (a, b, c) = 1. It may have many solutions if there exists a common factor among a, b, c and physically observed that it has many solutions.

Corollary:

It also covers the proof of FLT i.e. $a^n + b^n = c^n$ where n > 2 has no existence.

With the acceptance of proof towards Beal-conjecture we can review once again the trinomial relation $a^x + b^y = c^z$ where x, y, z > 2 & (a, b, c) = 1. It can be divided into two equations. The relation where anyone exponent is greater than two and other two are restricted to two can be said as M-equation whereas two exponents are greater than two and one is obviously restricted to two can be said as D-equation. The N-equation has already been defined as a trinomial relation where the zygote elements are of rational integer. For irrational zygote elements (either both or single) it is known as N_{ize}-equation. The N-equation produces both kinds of examples of M-equation & D-equation. Examples of M-equation under N-equation are found in abundance and needless to mention. One example of D-equation under N-equation can be given by $33^8 + 1549034^2 = 15613^3$. Now let us discuss different type of D-equations under N-equation.

3. How an even number with odd exponent produces a negative wing (difference of two squares of odd integers) confirming the proof $a^{2m} + b^{2n} = c^{2p}$ has no existence if m, n, p > 1

Any combination among $2^{2n-1}, d_1^{2n+1}, d_2^{2n+1}, \ldots\ldots, d_k^{2n+1}$ less than the rest by magnitude can produce such type of negative wing where all d_k stands for odd integer.

$\Rightarrow (2.\prod d_i. \prod d_j)^{2n+1} = (2^{2n-1}. \prod d_i^{2n+1} + \prod d_j^{2n+1})^2 - (2^{2n-1}.\prod d_i^{2n+1} - \prod d_j^{2n+1})^2$ where $\prod$ stands for continued product and $(d_i, d_j) = 1$ and total number of such wings = 2^k.

Product of any two such wings by N_d-operation can further produce two negative wings resulting an even number with odd exponent i.e. $(2D_1)^{2n+1}.(2D_2)^{2n+1} = (\alpha_1^2 - \beta_1^2)(\alpha_2^2 - \beta_2^2)$

$\Rightarrow (4D_1D_2)^{2n+1}$ = two negative wings by N_d-operation.

But in both the cases the negative wings will produce $2^{2\lambda}$ common factor and LHS reduces to

$2^{4n+2-2\lambda}.(D_1D_2)^{2n+1}$ where if we consider $2n + 1 = \mu(4n + 2 - 2\lambda)$ then $\mu = (2n + 1)/2(2n + 1 - \lambda)$ which is fraction and not accepted $\Rightarrow$ LHS is not in power form.

$\Rightarrow$ Any even number in the form of $(2^\lambda d)^{2n+1}$ where $\lambda > 1$, $n \geq 1$ fails to produce any negative wing.

Similarly, from N-equation if we take $(2^\lambda d)^2 = d_1^2 - d_2^2$ where $\lambda > 1$ & $d \in$ odd, we get

$(2^\lambda d)^4 = (d_1^2 + d_2^2)^2 - (2d_1d_2)^2$ & after cancellation 2^2 on both sides we lose the power 4 on LHS.

$\Rightarrow (2^\lambda d)^n$ where $\lambda > 1$ & $n > 2$ fails to produce any negative wing.

This confirms the fact that all the exponents cannot be even greater than 2.

It further confirms that if Beal equation exists the exponents of two odd elements will be either both even or combination of even & odd.

Some examples of D-equation produced by

$(2.d_1d_2)^{2n+1} = (2^{2n-1}.d_1^{2n+1} + d_2^{2n+1})^2 - (2^{2n-1}.d_1^{2n+1} \sim d_2^{2n+1})^2$ ………… **(C)** can be given below.

$(96222)^3 = 30042907^2 - 43^8$

$\Rightarrow (2.7.29.3.79)^3 = \{2.(7.29)^3 + (3.79)^3\}^2 - \{2.(7.29)^3 - (3.79)^3\}^2$ where n = 1

Let us take the special case $(2d)^{2n+1} = (d^{2n+1} + 2^{2n-1})^2 - (d^{2n+1} \sim 2^{2n-1})^2$

Here, $d^{2n+1} \pm 2^{2n-1}$ = either a square integer or powerless for n >1

$\Rightarrow$ All D-equations extracted from this are of the nature $(2d)^{2n+1} = d_1^2 - d_2^4$ or $= d_1^4 - d_2^2$

Examples: For n = 1, we have $(2d)^3 = (2 + d^3)^2 + (2 - d^3)^2$ where for d = 1, $2 - d^3 = 2 - 1^3 = 1^2$
$\Rightarrow (2.1)^3 = 3^2 - 1^4$
For n = 2, we have $(2d)^5 = (2^3 + d^5)^2 + (2^3 - d^5)^2$ where for d = 1, $2^3 + 1^5 = 3^2 \Rightarrow (2.1)^3 = 3^4 - 7^2$
For n = 1 & d = 3, we have $3^3 - 2^1 = 5^2 \Rightarrow (2.3)^3 = (2 + 3^3)^2 - (3^3 - 2)^2 \Rightarrow 6^3 = 29^2 - 5^4$ & so on.

4.

a) The expression $2^{2n-1}.d_1^{2n+1} + d_2^{2n+1}$ produces square integer when $d_2 \in 2^{nd}$ kind

b) The expression $2^{2n-1}.d_1^{2n+1} \sim d_2^{2n+1}$ produces sq. integer when $d_2 \in 1^{st}$ kind for n > 1

c) The expression $d_2^{2n+1} - 2^{2n-1}.d_1^{2n+1}$ produces sq. integer when $d_2 \in 1^{st}$ kind for n = 1

d) The expression $2^{2n-1}.d_1^{2n+1} - d_2^{2n+1}$ produces sq. integer when $d_2 \in 2^{nd}$ kind for n = 1

e) For n = 1, $2d_1^3 + d_2^3$ cannot produce any square integer.

Let us assume two square integers p^2 & q^2 received from the positive expression either both from n > 1 or both from n = 1 & then adding we get $2^k.d + d_2^{2n+1} + d_4^{2m+1} = p^2 + q^2$ where k > 1
Now, $p^2 + q^2$ is of the form $2(4x + 1)$
$\Rightarrow d_2^{2n+1}$ & d_4^{2m+1} both are of same kind $\Rightarrow d_2$ & d_4 both are of 2^{nd} kind as there exists two relations $2.0^3 + 1^3 = 1^2$ and $2^3.1^5 + 1^5 = 3^2$ where $1 \in 2^{nd}$ kind. In both the cases if they produce further square integers d_2 & d_4 must be of 2^{nd} kind or it will never produce any square integer.
Similarly, if we assume two square integers p^2 & q^2 received from the two negative expressions where n > 1 we will get d_2 & d_4 are of same kind. If we take p^2 from positive expression & q^2 from negative expression we will find d_2 & d_4 are of opposite kind. This confirms $d_2, d_4 \in 1^{st}$ kind for n > 1 and for negative expression.
For n = 1 and on the basis of same analysis we can say $d_2^3 - 2.d_1^3$ will produce many square integers when $d_2 \in 1^{st}$ kind as there exists a relation $3^3 - 2.1^3 = 5^2$ and $2.d_1^3 - d_2^3$ will produce many square integers when $d_2 \in 2^{nd}$ kind as there exists a relation $2.(7.29)^3 - (3.79)^3 = (1849)^2$
It confirms that $d_2^3 + 2.d_1^3$ fails to produce any square integer as it is contradictory.

5. Power characteristic of the expressions $|2^{2n-1}.d_1^{2n+1} \pm d_2^{2n+1}|$

Case I, $Y = 2^{2n-1}.d_1^{2n+1} + d_2^{2n+1}$ where n > 1, $d_2 > 1$
Say, $Y = p^{2m} = (2x - 1)^{2m}$ & $d_2^{2n+1} = (4y + 1)^{2n+1}$
$\Rightarrow 2^{2n-1}.d_1^{2n+1} = (2x - 1)^{2m} - (4y + 1)^{2n+1} = 2^k$(an odd integer) form where k > 1 after binomial expansion which can be accepted as it is matching with LHS for n > 1.
But $Y = p^{2m+1}$ is not acceptable as it is in the form of 2(odd integer), which is not matching with LHS for n > 1.
When p^{2m} is accepted m should not contain any odd factor, otherwise it will be of same nature of $p^{2m+1} \Rightarrow 2m$ must be in the form of 2^λ

Case II, For n = 1 & $d_2 = 1$ or $d_2 > 1$, if we analyze in the same way we will find that p^{2m+1} is not accepted and p^{2m} is accepted as $p^{2^\wedge\lambda}$.
Case III, Similarly for other two expressions i.e. $2^{2n-1}.d_1^{2n+1} \sim d_2^{2n+1}$ if we analyze in the same way we will get same result.
$\Rightarrow (2.d_1d_2)^{2n+1} = (2^{2n-1}.d_1^{2n+1} + d_2^{2n+1})^2 - (2^{2n-1}.d_1^{2n+1} \sim d_2^{2n+1})^2$
$= p^2 - q^{2^\wedge\lambda}$ form or $p^{2^\wedge\lambda} - q^2$ form (both cannot be in power form 2^λ simultaneously)
It confirms the fact that if Beal equation exists both the exponents of odd elements must be combination of odd & even whereas even element in the form of 2(odd) must contain odd exponent.

Example: $(96222)^3 = 30042907^2 - 43^8 = 30042907^2 - 43^{2\wedge 3}$

$\Rightarrow (96222)^3 = \{2.(7.29)^3 + (3.79)^3\}^2 - \{2.(7.29)^3 - (3.79)^3\}^2$

Let us take the special case $(2d)^{2n+1} = (d^{2n+1} + 2^{2n-1})^2 - (d^{2n+1} \sim 2^{2n-1})^2$

Here, $d^{2n+1} \pm 2^{2n-1}$ = either a square integer or powerless for n >1

$\Rightarrow$ All D-equations extracted from this are of the nature $(2d)^{2n+1} = d_1^2 - d_2^4$ or $= d_1^4 - d_2^2$

Examples: For n = 1, we have $(2d)^3 = (2 + d^3)^2 + (2 - d^3)^2$ where for d = 1, $2 - d^3 = 2 - 1^3 = 1^2$

$\Rightarrow (2.1)^3 = 3^2 - 1^4$

For n = 2, we have $(2d)^5 = (2^3 + d^5)^2 + (2^3 - d^5)^2$ where for d = 1, $2^3 + 1^5 = 3^2 \Rightarrow (2.1)^3 = 3^4 - 7^2$

For n = 1 & d = 3, we have $3^3 - 2^1 = 5^2 \Rightarrow (2.3)^3 = (2 + 3^3)^2 - (3^3 - 2)^2 \Rightarrow 6^3 = 29^2 - 5^4$ & so on.

6. How Power generates in D-equation under N-equation (A) & (B)

Let us consider Eq. (B), $a^2 + b^2 = c^n$ where a, c, n $\in$ odd integer

$\Rightarrow a = \alpha^n - {}^nc_2\alpha^{n-2}\beta^2 + \ldots\ldots + {}^nc_{n-1}\alpha\beta^{n-1}$ where α is odd & β is even integers.

$\Rightarrow a = \alpha^n - \alpha(B\alpha + C) = \alpha^n - \alpha A$ where obviously A is free from factor α

If 'a' is in power form with even exponent say $\alpha = p^{2m} \Rightarrow \alpha A = \alpha^n - p^{2m}$ where obviously p has a factor α

$\Rightarrow \alpha A = \alpha^n - \alpha^{2m}q^{2m}$ which will hold hood when and only when m = 1 & A contains α by virtue of n is prime and equal to α so that n becomes a common factor among all the binomial coefficients ${}^nc_2$, ${}^nc_4$, ……. ${}^nc_{n-1}$

$\Rightarrow$ n is odd prime and 'a' produces power as $\{(na)^2\}^2 = (na)^4$

Similarly, when 'a' produces odd power 'c' produces power as $(nc)^4$ provided n $\in$ 2nd kind prime. In both the cases n = α

If a $\in$ 1st kind with n $\in$ 2nd kind prime we must get D-relation like $a^n + (nc)^4 = b^2$ by Eq. (A –) i.e. considering negative sign. $\Rightarrow$ if n is prime the relation is $\alpha^2A = \alpha^n - (\alpha p)^2 \Rightarrow$ even power must be in the form of 2^λ

7. How Power generates in D-equation under N_{ize}-equation

Case I: When both exponents of odd elements are odd (Eq. A or B)

Here, the zygote elements α & β are the mix of irrational & rational nature.

As per previous, when 'c' produces odd power we can have the relation like $\alpha A = \alpha^n - p^{2m+1}$ or $\alpha A = p^{2m+1} - \alpha^n$ where p^{2m+1} must exist in the form of $(\alpha p)^{(2m+1)/2}$. Here α should be considered as $\alpha^{(2m+1)/2}$

Here, the power n need not to be a prime equal to α but in both the cases the element 'a' must be divisible by α. It will create a relation like $a^{2m+1} + b^2 = c^{2n+1}$

Similarly, when 'a' produces odd power we can get the relation like $a^{2n+1} \pm b^2 = \pm c^{2m+1}$ where c is divisible by α. In both the cases m may be equal to n also.

Case II: when both exponents of odd elements are mix of odd & even (w.r.to Eq. A+ & B)

As the even power is always in the form of 2^λ there cannot exist any irrational zygote element except at the initial stage i.e. $\alpha A = \alpha^n - (\sqrt{\alpha p})^2$, n $\neq$ prime

Case III: when both the exponents of odd & even element are odd (Eq. C)

In Eq. (A) or (B) when the odd elements 'a' or 'c' produces odd power we have the relation

$\alpha A = \alpha^n - p^{2m+1}$ which is always satisfied by $\alpha A = \alpha^n - (2p)^{2m+1}$ where the zygote element

$\alpha = 2^{(2m+1)/2}$. Here we can receive a relation like $a^2 + (2d)^{2m+1} = c^{2n+1}$ or, $a^{2n+1} + (2d)^{2m+1} = c^2$

i.e. Eq. (C) under N_{ize}-equation. (here α acts as an even zygote element β)

Case IV: when even element produces odd power along with even power of an odd element

Same logic as explained in case II will hold good with respect to Eq.(A+) & B

Case V: when both the odd elements are in power form and sum is a square of even integer

It happens as per Eq.(A-) i.e. $a^n + c^m = (2b)^2$ where Eq. (B) will not hold good. It is to be expanded with respect to a^n where both the zygote elements are irrational with nature as shown in example. But whatever may be the nature of zygote elements nature of even power will remain same i.e. 2^λ.

In view of the above we can say, all that exist are under N-equation except when two exponents are odd or mix lying on same side.

8. More authentic proof of Beal Conjecture

Let us review the case when all the elements are in power form beyond 2.

It has already been established that if this equation exists it will exist in the following two ways, basically derived from Pythagorean relation $a^2 + b^2 = c^2$ where a, c are odd integers.

i.e. either, $(\alpha^2 - \beta^2)^n \pm (2b_1)^{2r+1} = \pm (c_1\alpha)^{2^\wedge\lambda}$ or, $(a_1\alpha)^{2^\wedge\lambda} + (2b_1)^{2r+1} = (\alpha^2 + \beta^2)^n$

With respect to power formation 'n' that has produced power 2^λ it is proved that the zygote odd element 'α' must be odd prime and equal to 'n'.

Now, applying the same logic for the power formation of (2r + 1) with respect to power 'n' we can say 'b' must have a factor 'α' $\Rightarrow (a, b) = \alpha^k \Rightarrow (a, b, c) = \alpha^k$

$\Rightarrow$ Without accepting a common prime factor among a, b, c we cannot have a relation like

$a^x + b^y = c^z$ where x, y, z > 2.

Few examples:

$3^5 + 10^2 = 7^3$, with respect to 7^3 it can be compared to Eq. (B) for n = 3 which is satisfied by zygote elements $\alpha = 2$ & $\beta = \sqrt{3}$

But if it is compared to Eq. (A) with respect to 3^5 the zygote elements can be found out from

$(\alpha + \beta)^5 = 7^{3/2} + 10$ & $(\alpha - \beta)^5 = 7^{3/2} - 10$

$\Rightarrow \alpha = ½[\sqrt[5]{7\sqrt{7} + 10} + \sqrt[5]{7\sqrt{7} - 10}]$ & $\beta = ½[\sqrt[5]{7\sqrt{7} + 10} - \sqrt[5]{7\sqrt{7} - 10}]$

The example $420229^3 + 213978492^2 = 493237^3$ has the zygote elements $\alpha = 61\sqrt{61}$(consider as $\sqrt{61.61.61}$) & β = 516 with respect to 493237^3 when it is compared to equation (B). Here 420229 is divisible by 61.

Example $3^5 + 11^4 = 122^2$ actually falls under Eq.(A–) as $3 \in 1^{st}$ kind i.e. $3^5 - 122^2 = -11^{2^\wedge 2}$

Zygote elements are $\alpha = ½[\sqrt[5]{122 + 11^2} + \sqrt[5]{122 - 11^2}]$, $\beta = ½[\sqrt[5]{122 + 11^2} - \sqrt[5]{122 - 11^2}]$

$7^6 + 7^7 = 98^3$, $19^4 + 38^3 = 57^3$, etc.

Reference:

[1] Self, IJSER, May-edition, 2017, Vol-8, Issue-5, Theory of wings, page 133-148 where concerned page 137-138

[2] Self, IJSER, May 2014, vol-5, issue-5, Relevant page 178 (S. no. 5) regarding proof of FLT.

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